INVESTIGATING THE PROCESS OF THE STEADY EXTRUSION OF A COMPACTED MATERIAL

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In the compression of porous plastic materials (metals) it is usually a billet that has been precompacted that is subjected to treatment, this billet representing a compact body, although damaged (pores, microcracks), whose density may amount to 90-95% of the density of the solid phase. Extrusion begins when the pressure of the punch becomes sufficiently great. The geometric scheme of this process is shown in Fig. 1: 1) dye; 2) punch; 3) billet; 4) container. Steady flow is possible when the initial density ρ_0 of the material is so great that the pressure on the punch, required for compression of the material in the container, is greater than the pressure required for the onset of extrusion. In this case, extrustion will proceed without compaction in the container and the material reaching the entry to the dye will be of the same density, i.e., equal to ρ_0. The fact that such a situation is possible demonstrates the extrusion of an uncompacted material. Let us add that the process is not immediately established, but only after the material which filled the inlet to the dye begins to egress from the dye. The process of steady extrusion for a compacted material has been dealt with in a number of studies (for example, [1, 2]).

Yield Conditions. The simplest generalization of the Trask yield conditions on compacted materials is the condition which, in the space of the main stresses, corresponds to regular hexagonal pyramids with a common base lying on the deviator plane [3], and whose apices are on the hydrostatic axis (Fig. 2). We will write this yield condition in the following form:

$$|\sigma_i - \sigma_j|/(2\tau_s) + |\sigma|/p_s = 1.$$

Here σ_i and σ_j are the principal stresses; σ is an average stress; τ_s and p_s are the shearing yield points and the omnidirectional uniform compression, with τ_s and p_s representing the known functions of density ρ (ρ is the density of the material, referred to the density of the solid phase). An infinite pyramid with such bases was dealt with in [4].

Definition of Compression Density. We will assume that the dye is a truncated circular cone with a flare angle φ_0. We will use the spherical coordinate system r, φ, θ (see Fig. 1). Let v_r, v_φ, and v_θ be the projections of velocity onto the corresponding coordinate directions.

We will limit ourselves to the case in which a uniform material of density ρ_0 enters the dye, and we will assume further that the dye itself is immobile. Then, owing to symmetry about the axis of the matrix we can assume that $v_\theta = 0$. Since $v_\varphi = 0$ on the axis of symmetry for the dye and its walls, the nature of the flow will be close to a three-dimensional radial flow. Consequently, this will be satisfied, provided that the dye is not too short. We will further assume, as is usually done in analogous situations, that the influence of contact friction makes itself felt only near the walls of the dye, so that shear in the main mass of the material can be ignored. We will assume that the radial velocity is a function exclusively of the polar radius: $v_r = v(r)$.

In the free discharge of free-flowing materials through a conical funnel we observe the eddying of the flow, i.e., a loss of axial symmetry [5]. However, in the compression of compacted billets no such phenomena are observed. The hypothesis of axial flow symmetry is therefore validated.

The strain-rate components are: $\varepsilon_r = dv/dr$, $\varepsilon_\varphi = \varepsilon_\theta = v/r$. The remaining components are equal to zero. Since $v < 0$, we have $\varepsilon_\varphi < 0$, $\varepsilon_\theta < 0$. In order to determine the sign of ε_r, let us note that for an incompressible and, consequently for sufficiently dense com-

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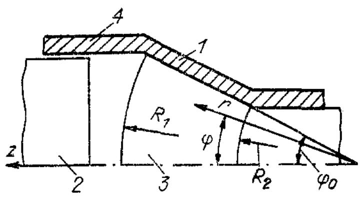

Fig. 1

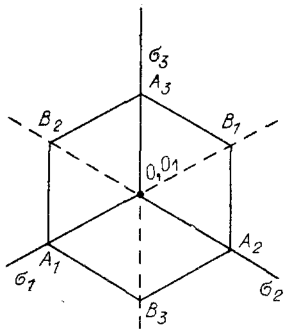

Fig. 2

pressed materials, $\varepsilon_r > 0$. These requirements are satisfied by the line O_1A_1 of the yield conditions (see Fig. 2), and the directions of r, φ, and θ correspond to 1, 2, and 3. The pyramid apex O_1 lies in the half space $\sigma < 0$.

We will write the equations for the line O_1A_1 in the form

$$\frac{\sigma_1-\sigma_2}{2\tau_s}-\frac{\sigma}{p_s}=1,\quad \frac{\sigma_1-\sigma_3}{2\tau_s}-\frac{\sigma}{p_s}=1. \tag{1}$$

We will assume in (1) that $\sigma_1 = \sigma_r$, $\sigma_2 = \sigma_\varphi$, and $\sigma_3 = \sigma_\theta$. It follows from (1) that $\sigma_\varphi = \sigma_\theta$, and thus

$$\sigma_r/\alpha - \sigma_\varphi/\beta = 1,\ \sigma_\varphi - \sigma_\theta = 0, \tag{2}$$

where

$$\alpha=\frac{6\tau_s p_s}{3p_s-2\tau_s},\quad \beta=\frac{6\tau_s p_s}{3p_s+4\tau_s}.$$

Applying the associated law of flow to (2), we obtain $\varepsilon_r = \lambda/\alpha$, $\varepsilon_\varphi = -\lambda/2\beta$. Eliminating λ from these equations, we come to the relationship $\varepsilon_r(3p_s + 4\tau_s) + 2\varepsilon_\varphi(3p_s - 2\tau_s) = 0$. When we substitute the expression for ε_θ and ε_φ, we have

$$(3p_s + 4\tau_s)(dv/dr) + 2(3p_s - 2\tau_s)(v/r) = 0. \tag{3}$$

We will assume $p_s=\frac{2}{\sqrt{3}}\frac{\rho^2 k}{(1-\rho)^{1/2}}$, $\tau_s = \rho^{3/2}k$ [6]. Equation (3) will then have the form

$$\begin{aligned}&(dv/dr)[\sqrt{3}\rho^{1/2} + 2(1-\rho)^{1/2}] +\\ &+ 2(v/r)[\sqrt{3}\rho^{1/2} - (1-\rho)^{1/2}] = 0.\end{aligned} \tag{4}$$

The continuity equation under these assumptions and on the strength of the steady-state nature of the flow represents $(v/\rho)(d\rho/dr) + dv/dr + 2v/r = 0$. It has the integral $\rho v r^2 = c_1$. The constant c_1 is determined from the conditions at the entry to the dye ($v = v_1$, $\rho = \rho_1$ for $r = R_1$): $c_1 = \rho_1 v_1 R_1^2$. Thus, $v = \rho_1 v_1 R_1^2/(\rho r^2)$. Substituting this v into (4), we derive the equation of relative density

$$\frac{dr}{r}+\frac{2}{3}g(\rho)\,d\rho=0\ \left(g(\rho)=\frac{\sqrt{3}\rho^{1/2}+2(1-\rho)^{1/2}}{4\rho(1-\rho)^{1/2}}\right). \tag{5}$$

The integral of this equation, satisfying the condition $\rho = \rho_1$ for $r = R_1$, is written as

$$\ln(r/R_1) + (2/3)G(\rho_1, \rho) = 0, \tag{6}$$

where

$$G(\rho_1,\rho)=\int_{\rho_1}^{\rho} g(\rho)\,d\rho=(\sqrt{3}/2)\,(\arcsin\rho^{1/2}-\arcsin\rho_1^{1/2})+(1/2)\ln(\rho/\rho_1).$$

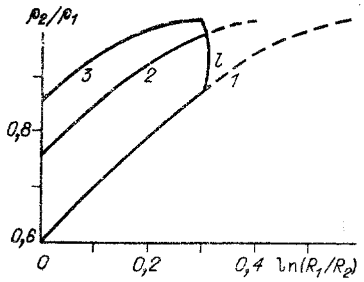

Fig. 3

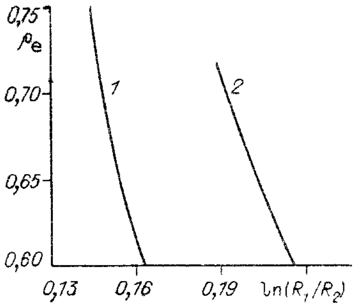

Fig. 4

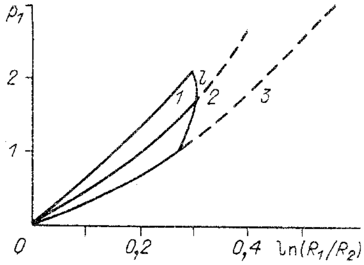

Fig. 5

We can see that with some $r = R_* > 0$ the particles of the material reach their limit density of $\rho = 1$. The case $R_2 < R_*$ requires special study and is not dealt with here, i.e., we assume that $R_2 \geq R_*$.

Assuming that $\rho = \rho_2$ for $r = R_2$ in (6), we derive the equation for the determination of density ρ_2 at the outlet from the die:

$$\ln(R_1/R_2) - (2/3)\, G\,(\rho_1,\ \rho_2) = 0. \tag{7}$$

Since we are dealing with a steady flow, ρ_1 = const.

In Fig. 3 (1-3: ρ_1 = 0.85, 0.75, and 0.6) we see the graphs for ρ_2/ρ_1 for various ρ_1 as functions of the magnitude of the reduction for which it has been assumed that $\ln(R_1/R_2)$.

Calculation of the Compression Pressure. We will compile the equilibrium equation in accordance with the Hill method [7]. The equation of the virtual powers under these assumptions with regard to the nature of the flow yields

$$\int_{R_1}^{R_2}\int_{0}^{\varphi_0}\left(r\frac{\partial\sigma_r}{\partial r}\sin\varphi + 2(\sigma_r - \sigma_\varphi)\sin\varphi + \frac{\partial}{\partial\varphi}(\tau_{r\varphi}\sin\ \varphi)\right)vr\,d\varphi\,dr = 0.$$

We will assume that the normal stresses depend exclusively on the polar radius r, and we will integrate over φ. Bearing in mind that v is an arbitrary function of r, we derive the equilibrium equation

$$d\sigma_r/dr + 2(\sigma_r - \sigma_\varphi)/r + (\tau_{r\varphi}/r)\,\mathrm{ctg}\,(\varphi_0/2) = 0. \tag{8}$$

The quantity $\tau_{r\varphi}$ is determined from the conditions of friction at the surface of the die. According to the Coulomb friction law $\tau_{r\varphi} = f|\sigma_\varphi| = -f\sigma_\varphi$. After substitution of this expression Eq. (8) assumes the form

$$r d\sigma_r/dr + 2\sigma_r - 2a\sigma_\varphi = 0,\ a = 1 + 0.5f\mathrm{ctg}(\varphi_0/2). \tag{9}$$

Expressing σ_φ in terms of σ_r from (2) and substituting into (9), we come to the differential equation for σ_r. Using (5), we make the transition from the independent variable r to the argument ρ. After transformation we derive the differential equation

$$\frac{d\sigma_r}{kd\rho} - h(\rho)\frac{\sigma_r}{k} = h_0(\rho), \tag{10}$$

where $h(\rho) = [a(\sqrt{3}\rho^{1/2} - (1 - \rho)^{1/2}) - (\sqrt{3}\rho^{1/2} + 2(1 - \rho)^{1/2})]/[3\rho(1 - \rho)^{1/2}]$, $h_0(\rho) = 2\rho a/[\sqrt{3}(1 - \rho)^{1/2}]$. Solution (10), satisfying condition $\sigma_r = 0$ for $r = R_2$, has the form

$$\frac{\sigma_r}{k} = \exp\ (H(\rho))\int_{\rho_2}^{\rho} h_0(\rho)\exp(-H(\rho))\,d\rho\ \left(H(\rho) = \int_{\rho_2}^{\rho} h(\rho)\,d\rho\right). \tag{11}$$

The pressure p_1 at the entry to the die, required for steady-state extrusion, is determined by means of formula (11) when $\rho = \rho_1$:

$$\frac{p_1}{k} = \exp(H(\rho_1))\int_{\rho_1}^{\rho_2} h_0(\rho)\exp(-H(\rho))\,d\rho. \tag{12}$$

Since ρ_1 is associated with ρ_2 by relationship (7), formula (12) expresses the pressure p_1 at the inlet to the die in terms of the material density ρ_1 at this same point. With steady-state extrusion the material coming into the entry of the die must exhibit the same density, and this is possible only with an adequately high initial density, when the compression stage in the container is absent. If we neglect the elastic strains of the material, in the absence of compression the pressure p beneath the punch is equal to the pressure p_1 at the inlet to the die. The value of p at which compression begins within the container is expressed in terms of the initial density [8] as

$$\frac{p}{k} = \frac{2\rho_0}{\sqrt{3}(1-\rho_0)^{1/2}}. \tag{13}$$

There may be no compression if p is larger than p_1, as determined from (12). The maximum value of the initial density ρ_e at which compression is present in the container satisfies the equation

$$\frac{2}{\sqrt{3}}\frac{\rho_e}{(1-\rho_e)^{1/2}} = \exp(H(\rho_e))\int_{\rho_e}^{\rho_2} h_0(\rho)\exp(-H(\rho))\,d\rho, \tag{14}$$

which is obtained if we equate the right-hand sides of formulas (12) and (13), substituting ρ_1 and ρ_0 in these by ρ_e. Equations (14) and (7), in which ρ_1 and ρ_e must also be replaced by ρ_e, define ρ_e as a function of $\ln(R_1/R_2)$.

The graphs of this function for $a = 2.72$ (curve 1) and 2.15 (curve 2) can be seen in Fig. 4. Figure 5 shows the graphs of the pressure at the inlet to the die as a function of reduction when $\rho_1 = 0.85$, 0.75, and 0.6 (curves 1-3, respectively) and $a = 2.72$.

This solution is valid only if $0 \le |\sigma| \le p_s$ [2]. Since the quantity $|\sigma|$ diminishes with increasing movement of the particle toward the outlet from the focus of the strain, while density increases, it is sufficient to test out the condition $p_1 \le p_s$ at the entry to the matrix. Its satisfaction depends on the initial density and the magnitude of reduction. The line ℓ in Figs. 3 and 5 determines the maximum magnitude of the reduction $\ln(R_1/R_2)$ as a function of the initial density.

The solid lines identify those segments of the graphs in which the above-cited solution is in force.

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A MATHEMATICAL MODEL OF THE PROCESSES OF FATIGUE WEAR AND DISINTEGRATION

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An analysis was conducted in [1, 2] into the behavior of the coefficients of stress intensity at the tips of subsurface cracks, located in an elastic overstressed half plane whose boundary is affected by normal and tangential contact stresses. These stress-intensity factors determine the development of cracks in an elastic medium and, thus, the fatigue quasibrittle destruction of bodies in contact with each other. Moreover, fatigue destruction of bodies depends on the level of material contamination and its resistance to crack formation.

In the present article we have laid out a statistical mathematical model for the processes of fatigue wear and disintegration, based on a study of a uniform mechanism for the development of fatigue cracks in quasibrittle materials.

1. The Suitability of Applying the Mechanics of Quasibrittle Destruction to the Study of Contact Fatigue. The main premise of the theory of fatigue destruction is the formation and the development of scattered microcracks, initiated by various defects (nonuniformities) in the material: microscopic pores, pitting, carbides, nonmetallic inclusions, etc. The process involved in the development of fatigue cracks around such defects is governed by the properties of the material and the stressed state of the material in the immediate vicinity of the defects, and this, in turn, depends on the normal and tangential stresses at the contact, as well as on residual stresses within the material.

The experimental and theoretical research [2-4] carried out to date enables us to isolate the fundamental factors characterizing fatigue destruction under loads which generate no significant plasticity phenomena in the material, and we have specific reference here to: normal and tangential contact stresses, residual stresses, the level of material contamination in the contact bodies and lubricants, the parameters of cyclical resistance to crack formation in the material, the structure of the material, etc.

Let us ascertain the possibility of utilizing the results obtained in the solution of contact problems for elastic bodies with cracks, based on the linear mechanics of quasibrittle destruction, insofar as this pertains to our study of the processes of contact fatigue.

1.1. Relative duration of crack generation and propagation phases. A variety of statements can be found in the literature, including those that are contradictory [5-7]. The assertion of the predominance of the generation phase, as a rule, is speculative in

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